Practice Final Exam

April 25, 2022

Problem 1. (a) Find the directional derivative of $f(x, y, z) = xy^2 + x^2z + yz^3$ at the point $(-1, 0, 1)$, in the direction given by the vector $\langle 1, 2, -2\rangle$.

(b) Find the tangent plane to the surface $xy^2 + x^2z + yz^3 = 1$ at the point $(-1, 0, 1)$.

Problem 2. Let D be the region between the circles $(x-1)^2 + y^2 = 1$ and $(x-2)^2 + y^2 = 4$, and above the x-axis (where $y \geq 0$). Evaluate the integral

$$\iint_D y dA.$$

Problem 3. Evaluate the integral $\int_C yzds$, where C is the line segment from $(1, 0, 2)$ to $(3, -1, 3)$.

Problem 4. Let C be the curve given by $\mathbf{r}(t) = \langle t^2, e^t, t^3\rangle$ from $t = 0$ to $t = 1$. Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}$ is the field

$$\mathbf{F}(x, y, z) = \langle yz\cos(xz), y + \sin(xz), xy\cos(xz) + \cos(z)\rangle$$

Problem 5. Let C go from $(1, -1)$ to $(1, 1)$ along the path $x = y^4$, and then back to $(1, -1)$ along the path $x = 2 - y^2$. Evaluate the integral

$$\int_C (e^{x+y} + \cos(x^2))dx + (e^{x+y} - 3x)dy$$

Problem 6. Let S be the surface given by $z = x^2 + y^2$, $1 \leq z \leq 2$, oriented downward. Evaluate the integral $\int \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle x + y, y, 1 + z\rangle$.

Problem 7. Let S be the surface given by $z = xy$, $x^2 + y^2 \leq 1$, oriented upward. Evaluate the integral $\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$, where $\mathbf{F}$ is the field

$$\mathbf{F}(x, y, z) = \left\langle y, e^{x^4}\sin(1 - x^2 - y^2), z\right\rangle.$$

Problem 8. Let S be the surface given by $x^2 + y^2 + z^2 = 4$, $x \geq 0$, oriented in the direction of the positive x-axis. Evaluate the integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}$ is the field

$$\mathbf{F}(x, y, z) = \left\langle e^{y^2+z^2}, 3y, e^{y^2-1}\right\rangle$$

Problem 9. Let E be the solid bounded by the surfaces $x^2 + z^2 = 2$, $y + z = 3$, $y = 0$. Evaluate the integral $\iiint_E z dV$.

Problem 10. Let $f(x, y) = 3x^2 + y^2 + 6xy + 8y$.

(a) Find and classify the critical points of f(x, y).

(b) Does the function f(x, y) have a global minimum? Justify your answer.

Problem 11. (a) Set up and DO NOT evaluate the following integral in the order $dxdydz$:

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} x^2 dz dy dx.$$

(b) Set up and DO NOT evaluate the following integral in spherical coordinates:

$$\int_{0}^{\sqrt{3}}\int_{0}^{\sqrt{3-x^2}}\int_{\sqrt{3x^2+3y^2}}^{3} y dz dy dx.$$